

2/19/10

→ $G_1(n, p)$ not always feasible in the real world.

GROWTH MODEL: WITH PREFERENTIAL ATTACHMENT:

Start with a single vertex and at each unit of time add a vertex.

→ How to add edges?

- Always add an edge to the currently selected vertex
- for the other vertex choose among other existing vertices based on their degree.

↳

- with probability δ , we add edge from vertex just added to a vertex selected with probability proportional to degree of vertex.
→ so, two probabilities of δ & prob of choosing other vertex.

→ Let $d_i(t)$ be degree of i^{th} vertex at time t .

$$\sum d_i(t) = 2\delta t = \text{normalizing factor.}$$

→ Probability of picking degree ' i ' is $\frac{d_i(t)}{2\delta t}$

$$\Rightarrow \frac{d_i(t)}{\delta t} = \frac{d_i(t)}{2\delta t} = \frac{d_i(t)}{2t} \rightarrow (\text{A})$$

$$d_i(t) = a \sqrt{t} \quad \{ \text{If we solve previous eqn. 3} \}$$

$$\frac{d_i}{dt} = \frac{a}{2} t^{-\frac{1}{2}}$$

substituting this value in (A).

$$\frac{\partial}{\partial t} d_i(t) = \frac{d_i(t)}{2t} = \frac{a \sqrt{t}}{2t} = \frac{a}{2} t^{-\frac{1}{2}}. \quad \left. \begin{array}{l} \text{verified value} \\ \text{of } \frac{d_i}{dt} \end{array} \right\}$$

Initial Condition:

$d_i(t)$ at $t=i$ equals 5.

$$d_i(i) = 5 \Rightarrow a i^{\frac{1}{2}} = 5 \Rightarrow a = 5 i^{-\frac{1}{2}}$$

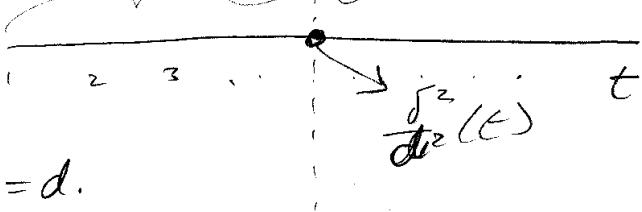
$$\therefore \boxed{d_i(t) = 5 \sqrt{\frac{t}{i}}}$$

expected degree $> d$

expected deg $< d$.

$$\text{If } i < \frac{5^2}{d^2} t$$

$$\text{then } d_i(t) \geq \sqrt{\frac{5d^2}{8t}} = d.$$

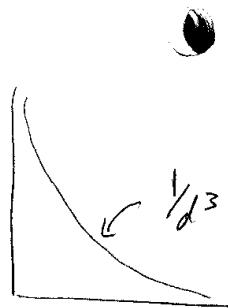


$$\therefore \boxed{d_i(t) \geq d}$$

d = constant value for
the degree of
graph.

$$\text{Prob}(\text{degree} < d) = 1 - \frac{\delta^2}{d^2}$$

$$\text{Prob}(\text{degree} = d) = \frac{2}{d} \left(1 - \frac{\delta^2}{d^2}\right) = \frac{2\delta^2}{d^3}$$



BRANCHING PROCESS:-

- Technique for growing a random tree.

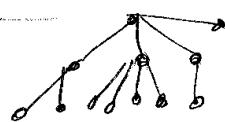
- Consider probability distribution for number of children:-

X (no. of children)	0	1	2	3
$P(X=x)$	p_0	p_1	p_2	p_3

→ Probability of extinction

→ Probability of immortality

In real graphs
the exponent of
 d lies between
2 & 3.



Applications in genetics, genome analysis, growth of species etc.

- This PMF is the same across all nodes of the random tree.

Consider a generating function:-

$$f(x) = p_0 + p_1 x + p_2 x^2 + p_3 x^3 + \dots$$

$$f(1) = p_0 + p_1 + p_2 + \dots = 1$$

$$f'(x) = p_1 + 2p_2 x + 3p_3 x^2 + 4p_4 x^3 + \dots$$

$$f'(1) = p_1 + 2p_2 + 3p_3 + 4p_4 + \dots = \text{Expected value of num. children.}$$

Let generating fn. for $y = f_y(x)$
 $\underline{z} = f_z(x)$

$y+z$ generating fn. for $y+z$ is:

$$f_{y+z}(x) = f_y(x)f_z(x)$$

Generating function for the first generation,

$$f_1(x) = f(x).$$

$$f_2(x) = f_1(f(x)).$$

$$\therefore f_{j+1}(x) = f_j(f(x)).$$

$$f_j(x) = \dots + \underbrace{c_i x^i}_{\text{prob}} + \dots$$

∴ prob of having 'i' children in j^{th} generation.

$$f_{j+1}(x) = f_j(f(x))$$

SOLUTIONS TO $f(x) = x$:

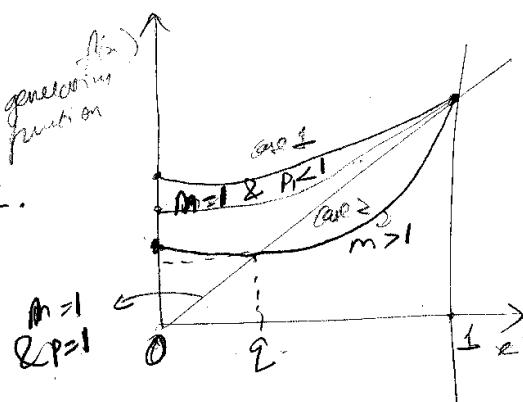
$$f(0) = p_0$$

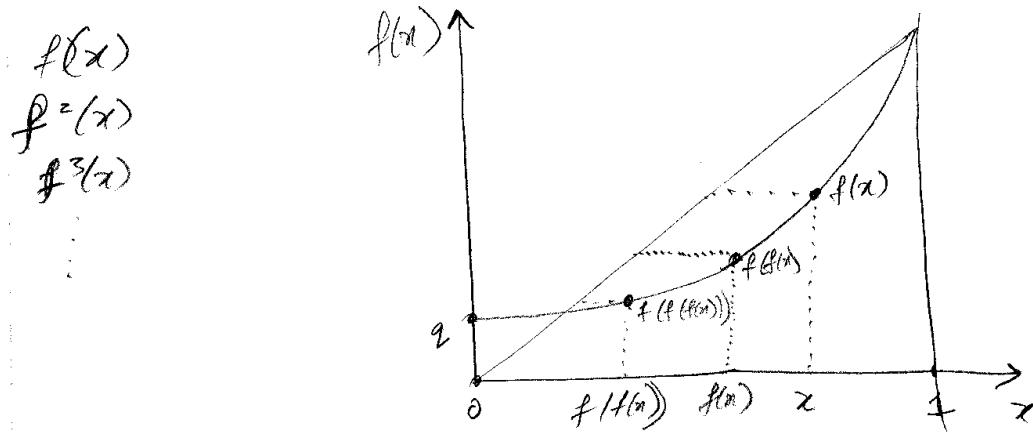
$$f(1) = 1$$

At some point, $m = f'(1)$, slope at $x=1$.

$$m=1 < p_1 < 1$$

$$p_1 = 1 \Rightarrow f(x) = x$$





$\therefore \lim_{j \rightarrow \infty} f^j(x) = q.$

$$f^*(x) = \lim_{j \rightarrow \infty} f^j(x) = q.$$

\Rightarrow There are 0^{**} children in j^{th} generation with probability q

\Rightarrow But probability $(1-q)$ is for infinite number of children.